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Network optimization for optimal product mix decisions in a graphite mining production process

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Received 18 December 2022
Revised 28 March 2023
1 June 2023
Accepted 3 July 2023

Abstract

Purpose – The purpose of this study is to develop a novel general mathematical model to find the optimal product mix of commercial graphite products, which has a complex production process with alternative sub-processes in the graphite mining production process.

Design/methodology/approach – The network optimization was adopted to model the complex graphite mining production process through the optimal allocation of raw graphite, byproducts, and saleable products with comparable sub-processes, which has different processing capacities and costs. The model was tested on a selected graphite manufacturing company, and the optimal graphite product mix was determined through the selection of the optimal production process. In addition, sensitivity and scenario analyses were carried out to accommodate uncertainties and to facilitate further managerial decisions.

Findings – The selected graphite mining company mines approximately 400 metric tons of raw graphite per month to produce ten types of graphite products. According to the optimum solution obtained, the company should produce only six graphite products to maximize its total profit. In addition, the study demonstrated how to reveal optimum managerial decisions based on optimum solutions.

Originality/value – This study has made a significant contribution to the graphite manufacturing industry by modeling the complex graphite mining production process with a network optimization technique that has yet to be addressed at this level of detail. The sensitivity and scenario analyses support for further managerial decisions.

Keywords Network optimization, Optimal product mix, Linear programming, Graphite mining, Mineral mixing problem

Paper type Research paper

1. Introduction

High-purity crystalline vein graphite is a commercial raw material (RM) that is in high demand, and used in many industries to produce a wide range of products. In the electronics industry, it is a key material in producing batteries and fuel cells, and it is also used as a lubricant in the metalworking and machinery industries. Vein graphite's high thermal and electrical conductivity make it an ideal material for thermal management systems and electronic devices. Additionally, vein graphite is used to produce advanced materials, such as graphene and carbon fiber, with a wide range of applications in the

International Journal of Industrial Engineering and Operations Management
Emerald Publishing Limited
e-ISSN: 2690-6104
p-ISSN: 2690-6090
DOI 10.1108/IJIEOM-12-2022-0072

aerospace, automotive and construction industries. Overall, the unique properties of high-purity crystalline vein graphite make it a versatile and valuable material for various commercial applications.

The mineral mixing problem is an optimization problem that arises in the mineral processing industry to meet precise quality and cost objectives (Braun, 1986). The goal is to blend various minerals using different production processes, resulting in different final products with the desired quality characteristics, such as purity, grade and particle size distribution. The problem is exacerbated by the fact that each mineral has different properties, including chemical and physical properties, and may require different processing methods. In solving the mineral mixing problem, various factors need to be considered, including the availability and cost of different minerals, the processing costs and times linked with each mineral, and the constraints associated with the processing plant's capacity. Additionally, specific quality requirements for the final product may be necessary, such as meeting certain environmental or regulatory standards. Mathematical optimization techniques such as linear, mixed-integer and nonlinear programming can be applied to solve the mineral mixing problem. These techniques involve formulating a mathematical model that represents the mineral processing system and its constraints, then using optimization algorithms to find the optimal solution that satisfies the desired objectives and constraints. On the other hand, when the manufacturing process has a network representation, network optimization techniques can be used to formulate mathematical models relatively easily. However, optimizing the production processes of mineral manufacturing companies according to the high demand in the international market to maximize profit is definitely an issue because of the complexity of the production process. Therefore, research on this "Mineral Mixing Problem" in the production process to find the optimum product mix is essential, and linear programming (LP) Techniques are applicable to these scenarios.

LP is a quantitative technique widely used in mathematical modeling to allocate limited resources to known activities to meet desired goals in different areas, such as production planning, resource allocation, inventory control and advertising (Dantzig, 2002). When formulating an LP model for a production process, the objective function and constraints are defined based on the specific requirements of the process. For instance, the objective function may be to maximize profit or minimize cost, while the constraints may include limits on the availability of RMs, production capacity, or quality standards. When the manufacturing process can be represented as a network model, it becomes relatively easy to transform it into a LP model. In this context, a network model refers to a representation of the manufacturing process that shows the flow of inputs and outputs between different stages, particularly to visualize the byproducts that are generated in different stages. When the production process is represented as a network model, it becomes easier to identify the variables and constraints that need to be included in the LP model. The nodes and arcs of the network can be used to represent the different stages of the process and the flow of materials and information between them (Glover *et al.*, 1992). Therefore, when the production process can be represented as a network model, LP model formulation is facilitated because the network structure provides a clear framework to identify the relevant variables and constraints.

Different case studies have used LP techniques to obtain managerial decisions such as allocations and usage of available production time, material and labor resources (Woubante, 2017; Willems *et al.*, 2019), find the optimum product mix (Moussa, 2021; Ezema and Amakom, 2012), to optimize condensing steam systems (Dragićević and Bojić, 2009) and to optimize of agricultural productivity (Igwe *et al.*, 2011; Sofi *et al.*, 2015). Nevertheless, research on the mineral mixing problem in the mining industry is scarce.

The optimization of the coal allocation procedure by Williams and Haley (1959), the determination of optimum crude oil input requirement by Adams and Griffin (1972) and the optimization of the RM allocation plan by Chanda (2018) are some examples of research studies that have been done previously in the area of interest. However, the studies conducted to find the optimum solutions to the mineral mixing problems by considering the whole complex production process with sensitivity analysis and scenario analysis, are scarce.

Therefore, this study aims to develop a novel general mathematical model for the mineral mixing problem in the graphite manufacturing industry with a theoretical contribution via a production network model to decide the optimum graphite mix which provides the maximum profit. Thereafter, the developed general model was used for a case study in a reputed graphite mining company in Sri Lanka engaged in natural vein graphite mining, processing and exports that extract approximately 300–400 metric tons of vein graphite per month. Current models rely on mineral mixing problems or other material mixing problems for the different manufacturing processes in various sectors, which may not accurately capture the complexity of the production process to obtain the optimal solutions, especially to make the suitable decisions regarding the byproducts. By incorporating data on product information, manufacturing processes, byproducts and constraints, this model has the potential to capture the optimum managerial decisions needed to optimize the objectives. The practical novelty and contributions of this research lie in its potential to improve complex manufacturing processes, especially if there are some byproducts, ultimately reducing time and material wastage and maximizing the total profit of the organization.

The rest of the paper is structured as follows: Section 2 reviews the literature related to the mineral mixing problem. Section 3 explains the research methodology, including the general model, while Section 4 illustrates the model formulation for the production process of the case study to test the proposed general model. Section 5 discusses the analysis and results of the case study, including sensitivity analysis and scenario analysis. Section 6 contains an overall discussion of the research work, with a summary of the analysis. Section 7 concludes the paper with further improvements.

2. Literature review

The LP technique has wide applicability to different management decision-making processes. Most literature in economic development supports the view that LP is a practical tool of analysis in allocating scarce resources to their optimal use, and it is of vital importance to the economies of underdeveloped countries. LP models can be solved using the simplex method that George Dantzig introduced in 1947 (Dantzig, 1951). LP can be effectively applied to product mix problems to find the optimal product mix in different manufacturing areas. In addition, it can identify the changes in the optimal solution if the model has parameters and constraints changes, which will help to optimize the production process further to make managerial decisions easier (Vakilifard *et al.*, 2013). For instance, the profit of apparel manufacturing depends mainly on the proper allocation and usage of available production time, material, and labor resources. Woubante (2017) developed an LP model to optimize the product mix for apparel manufacturing. Moreover, the study verified that the company's profit could be improved by 7.22% if the management used the optimal solution of the LP formulation. Moussa (2021) also proposed a new approach to LP optimization based on the Kubelka-Munk and Duncan theories to solve the textile color formulation problem, where the principle aimed to find the appropriate amount of dyes that needed to be mixed and the exact concentrations required to produce the desired color. The study identified that the proposed model provided accurate results with minimal error values and color differences. Further, the LP techniques can be effectively used to allocate the available resources for staff training; Fagoyinbo and Ajibode (2010) proposed a model to be used in the area of personnel management to minimize the staff training cost. Since energy and energy processing equipment costs have increased, energy management is necessary to develop efficient energy systems. Dragićević and Bojić (2009) proposed a model to minimize the total energy costs of steam condensing systems.

The LP technique uses resource allocation in production planning to increase agricultural productivity. Igwe *et al.* (2011) formulated a mathematical model for the fishing industry's semi-commercial farmers in the Ohafia Agricultural Zone to determine the optimum enterprise combination. The study concluded that the model would help to enhance food security among rural farmers in the study area. Besides, optimal RM allocation can lead to significant process improvement. Willems *et al.* (2019) proposed an optimization model to find the optimal allocation of potatoes, end products and manufacturing lines in a potato product manufacturing company with available data and predicted data using the nearest neighbor interpolation and random forest algorithms. The proposed LP approach and recovery prediction indicated potential savings in RM use and purchasing costs. Also, Sofi *et al.* (2015) used the LP technique to find the optimum resource allocation in the agricultural production of food crops and concluded that the proposed LP model is appropriate for finding the optimal land allocation for the major food crops. Some research studies were carried out on the bakery industry to allocate RMs and find the optimum product mix to satisfy the demand for bakery products and acquire the highest profit (Oladejo *et al.* (2019), Akpan and Iwok (2016)). Furthermore, some production planning problems can be implemented with LP techniques to utilize the limited available resources to satisfy the demand. Solaja *et al.* (2019) formulated an LP model for a production planning problem in a feed mill-producing company, where the optimum solution of the model improved the profit by streamlining the product range and cutting off the less productive products. Moreover, Ezema and Amakom (2012) implemented a LP model to find the optimal product mix of a productive firm in the layout, where the study concluded that only two sizes of the total eight "PVC" pipes should be produced to maximize the total profit.

In relation to the mining industry, research done on product mix problems (mineral mixing problems) is sparse. Williams and Haley (1959) proposed a mathematical model to optimize the coal allocation procedure. The study reduced transportation and a considerable amount of computation. A significant advantage of the study is that the approximations enabled engineers to devise the part of the coal allocation procedure that the local staff could operate. Adams and Griffin (1972) formulated a LP model to describe cost minimization in the U.S. petroleum refining industry and to determine crude oil input requirements, the output of byproducts, and capacity utilization. In addition, Chanda (2018) developed a Network LP formulation of the production-planning problem for a mining and metallurgical complex. The developed mathematical model provided the optimal production plan with the RM allocation to minimize production and distribution costs.

Previously, most studies were carried out to find the optimum product mix or production plan to maximize the total profit or minimize the total cost. However, those studies were not done with a view to finding the details of byproducts and byproduct operations and to analyzing the production process with sensitivity and scenario analysis for further managerial decisions. Indeed, the optimization techniques used for the graphite mining production process are virtually inaccessible. In particular, there is an absence of Network LP models that analyze production distribution type systems in the mining industry. Therefore, this paper addressed the above mentioned lacunas and developed a network LP model to find the optimal product mix for the mineral mixing problem based on the graphite mining production process.

3. Methodology

Network models are a significant part of special structures in LP, where they can be used to represent complex production processes. Network models allow for the transformation of a production process into a mathematical model, which can be beneficial when the process is complicated and challenging. On the other hand, LP is a widely recognized operational technique that is designed to solve mathematical models with linear objective and constraint functions. Besides, suppose a production process can be mapped as a network and implemented as a LP model to determine the optimal solution. In that case, it enables businesses to optimize their production processes. Furthermore, network models provide a useful tool for analyzing and optimizing a wide range of other complex systems beyond production processes. Therefore, this approach can be used for mineral mixing problems since it is also a complex production process that needs to maximize efficiency and profit. The overall modeling process adopted for the mineral mixing problem depends on different factors, and they can be discussed as follows. Accordingly, deciding the product mix of a graphite manufacturing company is formulated as an LP problem by identifying system requirements in terms of organizational objectives, constraints and conditions that exist and that can be converted into a researchable format.

The overall modeling process adopted for the mineral mixing problem depends on different factors, and it can be discussed as follows. Accordingly, deciding the product mix of a graphite manufacturing company is formulated as an LP problem by identifying system requirements in terms of organizational objectives, constraints and conditions that exist and converted into a researchable format. The steps of a modeling process can be depicted as a flow chart which is illustrated in Figure 1. The first step is problem formulation, which identifies the user attributes and needs and states the problem in a researchable way. The second step is to make assumptions because checking model assumptions is necessary before building a model that will be used for prediction. After that, according to the data and assumptions, a mathematical model is developed and solved using a solvable approach. Finally, if the solutions are acceptable, sensitivity and/or scenario analyses are performed to make recommendations. On the other hand, if the solutions are not acceptable, the assumptions are revisited because if assumptions are not met, the model may inaccurately reflect the data and likely to result in inaccurate predictions. Then the mathematical model is rebuilt.

Graphite manufacturing companies use natural graphite as run-of-mine from their underground mines and produce various graphite products by varying the carbon content, particle size and other physical properties. The composition of run-of-mine from underground mines varies for many reasons, including poor vein thickness, poor separation of minerals and rocks and minor carbon percentage enriched graphite. Moreover, to optimize the production process in order to find the optimum product mix, and formulate a general model, graphite manufacturing companies need to consider the following factors: demands, RM availabilities, output percentages, production operations, time requirements for each production operation, available machine hours, RM transformations, SPs and byproducts that are produced in each operation and minimum production requirements.

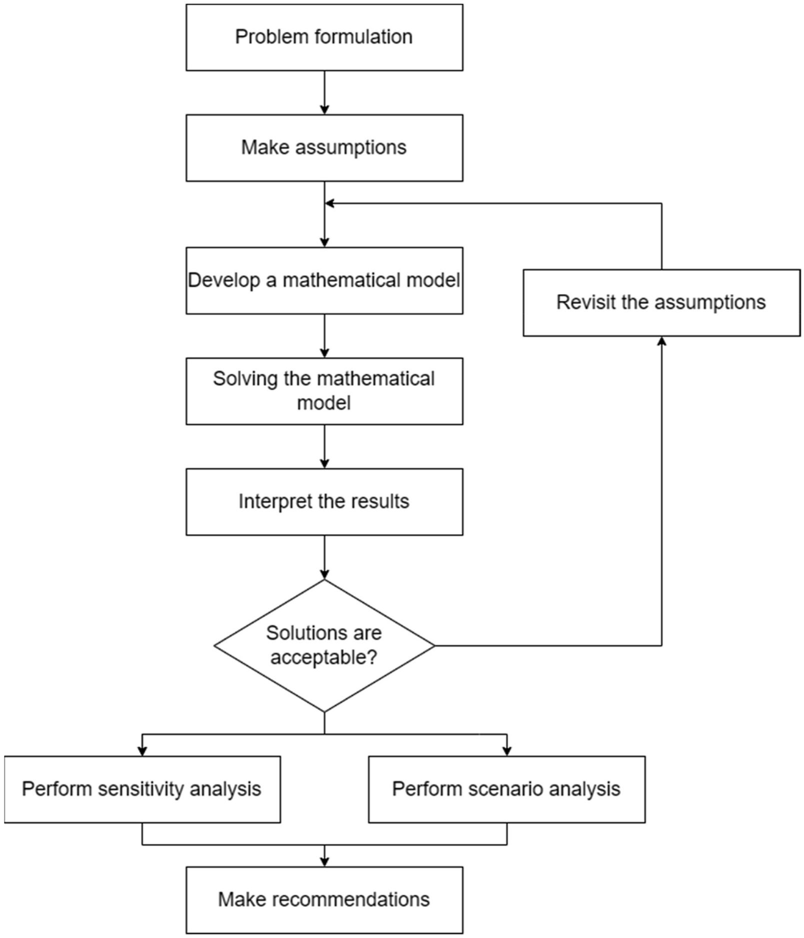


Figure 1.
Modeling process

Source(s): Authors' own work

The objective function of the general mathematical model is to maximize the total profit provided by the optimum graphite product mix. Besides, four constraints were generally identified for the formulation: RMs' availability, yields of the products and byproducts, demand satisfaction and machine hour availability. Therefore, by considering the general production process in graphite manufacturing companies, a general model can be formulated to optimize the manufacturing system to maximize the total profit and to find the optimal product mix with respect to the following parameters and available data. Let,

$i = Raw\ material;\ i = 1, 2, \ldots, n$
$j = Byproduct;\ j = 1, 2, \ldots, m$
$k = Saleable\ product;\ k = 1, 2, \ldots, l$
$h = Raw\ material\ that\ can\ be\ transformed\ into\ another\ raw\ material\ type;\ h \subseteq i$
$r = Operation\ type/plant;\ r = 1, 2, \ldots, s$
$Q_k = Amount\ of\ saleable\ product\ k\ should\ produce\ (in\ metric\ tons/month)$
$R_k = Revenue\ of\ product\ k\ per\ metric\ ton$
$C_k = Cost\ of\ product\ k\ per\ metric\ ton$
$a_i = Availability\ of\ mined\ raw\ material\ i\ quantity\ (in\ metric\ tons/month)$
$a_{hi} = Availability\ of\ RM\ h\ that\ can\ be\ used\ to\ transform\ into\ another\ RM\ i\ (in\ metric\ tons/month)$
$A_i = Availability\ of\ raw\ material\ i\ quantity\ for\ the\ production\ (in\ metric\ tons/month)$
$D_k = Demand\ of\ product\ k\ (in\ metric\ tons/month)$
$T_r = Time\ availability\ in\ plant\ k\ (in\ hours/month)$
$t_j = Time\ required\ to\ produce\ one\ metric\ ton\ of\ byproduct\ j\ (in\ hours)$
$t_k = Time\ required\ to\ produce\ one\ metric\ ton\ of\ saleable\ product\ k\ (in\ hours)$
$P_{ij} = Amount\ of\ raw\ material\ i\ required\ to\ produce\ the\ byproduct\ j\ (in\ metric\ tons)$
$P_{ik} = Amount\ of\ raw\ material\ i\ required\ to\ produce\ the\ saleable\ product\ k\ (in\ metric\ tons)$
$P_{jk} = Amount\ of\ byproduct\ j\ required\ to\ produce\ the\ saleable\ product\ k\ (in\ metric\ tons)$
$P_{kj} = Amount\ of\ saleable\ product\ k\ required\ to\ produce\ the\ byproduct\ j\ (in\ metric\ tons)$
$P_{kk} = Amount\ of\ saleable\ product\ (SP)\ k\ required\ to\ produce\ another\ SP\ k\ in\ metric\ tons\ (P_{ij}|i \neq j)$
$P_{hi} = Amount\ of\ raw\ material\ h\ that\ should\ be\ transformed\ into\ raw\ material\ i\ (in\ metric\ tons)$
$M_k = Minimum\ production\ requirement\ of\ product\ k$
$\alpha_{ij} = Output\ percentage\ (yield)\ for\ the\ production\ of\ byproduct\ j\ by\ product\ i$
$\beta_{ik} = Output\ percentage\ (yield)\ for\ the\ production\ of\ saleable\ product\ k\ by\ product\ i$
$\gamma_{jk} = Output\ percentage\ (yield)\ for\ the\ production\ of\ saleable\ product\ k\ by\ byproduct\ j$
$\delta_{kj\ or\ kk} = Output\ percentage\ (yield)\ for\ the\ production\ of\ byproduct\ j\ or\ SP\ k\ by\ SP\ k$

Therefore, the general LP model formulation for the mineral mixing problem can be stated as follows:

This problem aims to maximize the total profit (Profit = Revenue − Cost = $R_k - C_k$). Therefore, the objective of the LP model can be defined as: $Maximize\ Z = \sum_{\forall k}(R_k - C_k)Q_k$

The constraints of this mineral mixing problem can be formulated as follows.

Some selected mined RMs can be transformed into another RM type, i.e. $\sum_{\forall h} P_{hi} \leq \sum_{\forall h} a_{hi}\ \forall i$

The availability of each RM for the production process should equal the sum of mined RMs and transformed RMs minus the RMs used for transformation into other products.

$$\text{i.e. } A_i = a_i + \sum_{\forall h} P_{hi} - \sum_{\forall h} P_{ih}\ \forall i$$

The sum of RMs used to produce byproducts and SPs should be less than or equal to the availability of each RM, i.e. $\sum_{\forall j} P_{ij} + \sum_{\forall k} P_{ik} \leq A_i\ \forall i$

However, since some operations produce SPs and byproducts together, the RM weight requirement is the same for both SPs and byproducts for those operations.

$$\text{i.e. } (P_{ij})_r = (P_{ik})_r\ for\ some\ r$$

In addition, all produced byproducts should be used to produce SPs in the production process.

$$\text{i.e. } \sum_{\forall r}\left(\sum_{\forall i} \alpha_{ij} P_{ij}\right) = \sum_{\forall k} P_{jk}\ \forall j$$

Since some SPs can also be converted into other SPs, the SP weight, according to the demand, should produce equal to the sum of weights used from RMs and byproducts with their yields minus the sum of weights used from SPs with their yields for each manufacturing operation, i.e. $\sum_{\forall r}\left(\sum_{\forall i}\beta_{ik}P_{ik} + \sum_{\forall j}\gamma_{jk}P_{jk} - \sum_{\forall k}\delta_{kj}P_{kj} - \sum_{\forall k}\delta_{kk}P_{kk}\right) = Q_k\ \forall k$

The SPs produced according to the demand should be less than or equal to the demand, i.e. $Q_k \leq D_k\ \forall k$

The total time required to produce different products should be less than or equal to the available time for each manufacturing operation, i.e. $(t_j(P_{ij} + P_{kj}) + t_k\ (P_{ik} + P_{jk}))_r \leq T_r\ for\ each\ r$

According to the management decisions, minimum production requirements for each SP could exist, i.e. $Q_k \geq M_k\ \forall k$

Therefore, the general LP model can be stated as follows:

$$Maximize\ Z = \sum_{\forall k}(R_k - C_k)Q_k$$

$$Subject\ to:$$

$$\sum_{\forall h}P_{hi} \leq \sum_{\forall h}a_{hi}\ \forall i\ (RMs\ can\ be\ transformed\ into\ another\ raw\ material\ type)$$

$$A_i = a_i + \sum_{\forall h}P_{hi} - \sum_{\forall h}P_{ih}\ \forall i\ (Availability\ of\ raw\ material\ i\ for\ the\ production)$$

$$\sum_{\forall j}P_{ij} + \sum_{\forall k}P_{ik} \leq A_i\ \forall i\ (Supply\ constraints)$$

$$(P_{ij})_r = (P_{ik})_r\ for\ some\ r\ (Some\ operations\ produce\ both\ saleable\ and\ by\ products)$$

$$\sum_{\forall r}\left(\sum_{\forall i}\alpha_{ij}P_{ij}\right) = \sum_{\forall k}P_{jk}\ \forall j\ (All\ the\ produced\ byproducts\ should\ be\ used\ to\ produce\ saleable\ products)$$

$$\sum_{\forall r}\left(\sum_{\forall i}\beta_{ik}P_{ik} + \sum_{\forall j}\gamma_{jk}P_{jk} - \sum_{\forall k}\delta_{kj}P_{kj} - \sum_{\forall k}\delta_{kk}P_{kk}\right) = Q_k\ \forall k\ (Saleable\ products)$$

$$Q_k \leq D_k\ \forall k\ (Demand\ constraints)$$

$$(t_j(P_{ij} + P_{kj}) + t_k(P_{ik} + P_{jk}))_r \leq T_r\ for\ each\ r\ (Time\ constraints)$$

$$Q_k \geq M_k\ \forall k\ (Minimum\ production\ requrements)$$

$$P_{ij}, P_{ik}, P_{jk}, P_{kj}, P_{kk}, P_{hi}, Q_k \geq 0\ (Non-negativity\ condition)$$

The formulated general LP model above can be tested as a case study on a selected graphite manufacturing company. The case study approach allows for a detailed and comprehensive exploration of the research problem regarding the mineral mixing problem in the graphite manufacturing industry, where it enables researchers to examine the manufacturing processes involved, mineral compositions, and different factors with detailed data and to gain a detailed understanding of the specific challenges faced by the graphite manufacturing industry.

4. Case study

Sri Lanka is one of the major countries that extracts and produces commercially viable quantities of high-purity crystalline vein graphite with more than 98% carbon purity, which plays a pivotal role in the international market (Safshath *et al.*, 2021). Since Sri Lankan natural graphite is unique, Japan, Germany, the USA, India, Pakistan, Thailand, South Korea, Australia, the United Kingdom and China pose the highest demand in the international market. Natural vein graphite extracted underground as a run-off mine is subjected to various processing activities to meet applicable customer requirements and specifications before export. However, the Sri Lankan natural graphite industry cannot fulfill the demand due to the lack of or limited availability of other resources. Moreover, the monthly sales pattern differs and depends on global industrial and production trends in the automobile, high-tech, electrical and electronic industries.

In brief, the initial step taken by the mineral manufacturing company considered for the case study is mining raw graphite, consisting of different carbon contents in the form of lumps, chips, and powder used to produce RMs. The production process of the RMs is done by hand sorting or mechanical separation (crushing, sieving) methods according to the categorization of carbon content and size. Then, the RMs are used to produce SPs via Flotation, K&B, Rotex Screen and Ball Mill plants. During the graphite processing stage, it is a real dilemma to decide which graphite products through which production processes should be produced with available raw graphite using other resources in which the carbon content varies from 70% to 99%, as each type has distinct profit margins. Therefore, it is worth discovering which product mix yields the highest profit by utilizing the limited monthly underground mine production amounts with various realistic constraints.

According to the general model, a complete LP model can be developed for the case study in the selected graphite manufacturing company by considering the production network model and the available data. The decision variables are the amount of each SP that should be produced to maximize the total profit and the amount of RMs/byproducts/SPs required to produce byproducts/SPs for each operation. In addition, the constraints include RM extraction availability, demand and machine processing time. Figure 2 depicts the schematic diagram of the production flow of the relevant graphite mining company.

The assumptions and mathematical formulation of this mineral mixing problem can be stated as follows.

Assumptions:

(1) The average monthly production of raw graphite (Run-of-Mine) is 400 metric tons

(2) All the machines are in good condition yielding the expected level of performance

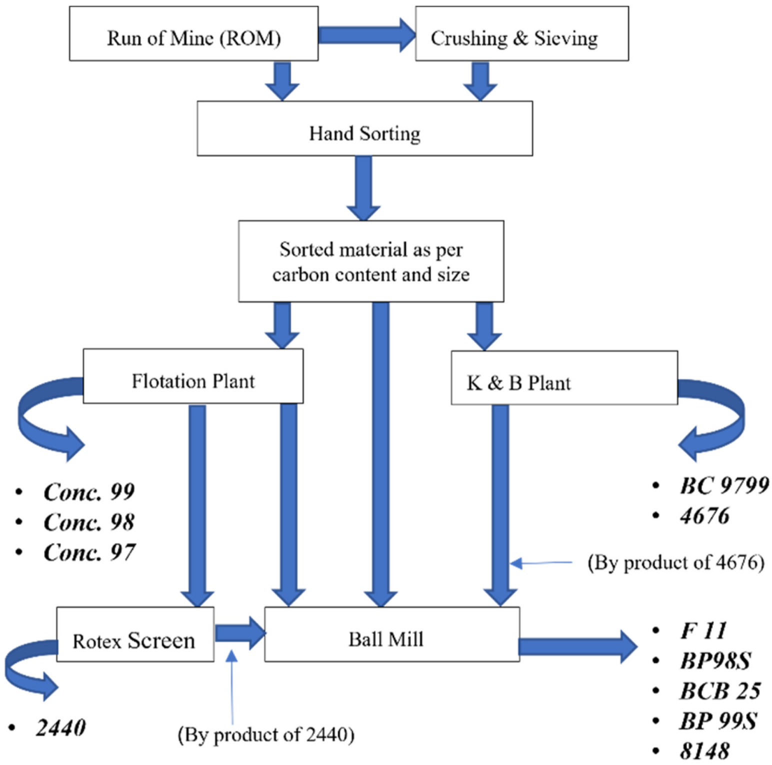
(3) There is no labor shortage

Objective function: The graphite mine produces ten types of different merchantable graphite grades, denoted by X_i where $i = 1, \ldots, 10$. Each trading graphite grade has a different profit margin, where the price of each graphite grade is defined based on its carbon percentage and the particle size of the RMs, production time and labor force by a cost analysis. Table 1 shows the notations used to identify the product information of each product, and the profit of a given product i is the difference between the revenue and the cost, defined as $R_i - C_i$.

Constraints: Three types of constraints have been identified: RM, machine hours and demand.

I. Constraints related to RM

RMs used to manufacture the demanded items were extracted with different carbon percentages, where Table 2 provides the notations and information related to the graphite



Source(s): Authors' own work

Figure 2.
Schematic diagram of the production flow in the company

Table 1.
Product information

Product name	Mathematical notation of the product	Demand in Euros/Month	Cost in Euros/MT	Revenue in Euros/MT	Machine hours required/MT
F 11	X_1	D_{X_1}	C_1	R_1	t_{X_1}
2440	X_2	D_{X_2}	C_2	R_2	t_{X_2}
BC 9799	X_3	D_{X_3}	C_3	R_3	t_{X_3}
BP 99 S	X_4	D_{X_4}	C_4	R_4	t_{X_4}
BP 98 S	X_5	D_{X_5}	C_5	R_5	t_{X_5}
BCB 25	X_6	D_{X_6}	C_6	R_6	t_{X_6}
8148	X_7	D_{X_7}	C_7	R_7	t_{X_7}
4676	X_8	D_{X_8}	C_8	R_8	t_{X_8}
Conc. 99	X_9	D_{X_9}	C_9	R_9	t_{X_9}
Conc. 98	X_{10}	$D_{X_{10}}$	C_{10}	R_{10}	$t_{X_{10}}$

Source(s): Authors' own work

mined in a month. In addition, RMs with a chemical composition of carbon below the percentage of 70% are not used for the production processes.

Furthermore, any byproduct generated while producing a salable or demand grade product can be used to produce other salable or demand grade products. It is a tremendous advantage to minimize the weight of byproducts because they are otherwise considered waste or unwanted material. Table 3 summarizes the details of byproducts.

II. Constraints related to machining hours

Each graphite product has a different production process, and Table 4 illustrates the processing methods of each product grade used for converting RM into a final or byproduct

Notation of the raw material	Form of the raw material	Carbon percentage	Mined quantity/month
Y_1	Lumps	99+	A_{Y_1}
Y_2	Lumps	97–99	A_{Y_2}
Y_3	Lumps/chips	90–97	A_{Y_3}
Y_4	Tub dust	70–90	A_{Y_4}
Y_5	Tub dust	Below 70	A_{Y_5}
Y_6	Pure rock	0	A_{Y_6}

Source(s): Authors' own work

Table 2. Raw material information

Notation	Form of the byproduct	Carbon percentage (%)	Machine hours required/MT
Z_1	Powder	97–99	t_{Z_1}
Z_2	Powder	99	t_{Z_2}
Z_3	Powder	97	t_{Z_3}

Source(s): Authors' own work

Table 3. Information about the byproducts

Raw material/ byproduct Name	Notation	Process applied	Final/byproduct Name	Notation	Output (%)	Name	Notation	Output (%)
Lumps 97 99	Y_2	K & B Plant	BC 9799	X_3	100%			
Lumps 97 99	Y_2	K & B Plant	4676	X_8	66.67%	9799 dust	Z_1	33.33%
Lumps 97 99	Y_2	Flotation	Conc.99	X_9	88.24%			
Lumps 90 97	Y_3	Flotation	Conc.99	X_9	70.31%			
9799 dust	Z_1	Flotation	Conc.99	X_9	80%			
9799 dust	Z_1	Ball mill	F 11	X_1	100%			
Conc. 99	X_9	Rotex screen	2440	X_2	71.43%	99powder	Z_2	28.57%
99 powder	Z_2	Ball mill	F 11	X_1	100%			
Conc. 99	X_9	Ball mill	BP 99S	X_4	100%			
Lumps 90 97	Y_3	Flotation	Conc.98	X_{10}	70.31%			
Conc. 98	X_{10}	Ball mill	BP 98S	X_5	100%			
Conc. 98	X_{10}	Ball mill	8148	X_7	100%			
Conc. 98	X_{10}	Ball mill	BCB 25	X_6	100%			
Tubdust 70–90	Y_4	Flotation	Conc.97	Z_3	62.5%			
Conc. 97	Z_3	Ball mill	F 11	X_1	100%			

Source(s): Authors' own work

Table 4. Processing methods

and the percentage of output from the input material. Here, the mined Lumps 99+ quantity are taken as Lumps 97–99 for the production process without further processing.

On the other hand, the time taken to process each graphite product varies in each mill or machine. In addition, the available machine hours of each mill and machine can be different. Table 5 exhibits the notations for each mill/machine's available machine hours in the processing department.

III. Constraints related to demand

Demand for each grade in each month can be identified from customers' orders in advance, where the demand pattern for graphite grades varies. Table 1 reveals the demand for each graphite product with its profit margins.

The overall production process of the company can be interpreted as a network described in Figure 3, to obtain a general idea of the process.

Therefore, the general LP for the mineral mixing problem can be set down as follows:

$$\text{Here}, i = y_1 = Lump\,99+, i = y_2 = Lump\,97-99, i = y_3 = Lump\,90-97, i = y_4 = Tub\,dust\,70-90,$$

$$h = y_1 = Lump\,99+,$$

$$j = z_1 = 9799\,dust, j = z_2 = 99\,powder, j = z_3 = Conc.\,97$$

$$k = x_1 = F11, k = x_2 = 2440, k = x_3 = BC9799, k = x_4 = BP99S, k = x_5 = BP98S, k = x_6 = BCB25S, k = x_7 = 8148, k = x_8 = 4676, k = x_9 = Conc.\,99, k = x_{10} = Conc.\,98,$$

$$and\, r = 1 = K\&B\,plant, r = 2 = Flotation, r = 3 = Ball\,mill, r = 4 = Rotex\,screen$$

The objective of this problem is to maximize the total profit. Therefore, the objective of the LP model can be defined as:

$$Maximize\,Z = \sum_{k=1}^{10}(R_k - C_k)Q_k \quad (01)$$

The constraints of this LP model can be formulated as follows.

The Lump 99+ RM used to produce Lump 97–99 by using a RM transformation process is less than the availability of mined Lump 99+ (a_{y_1}), i.e.

$$P_{y_1 y_2} \leq a_{y_1} \quad (02)$$

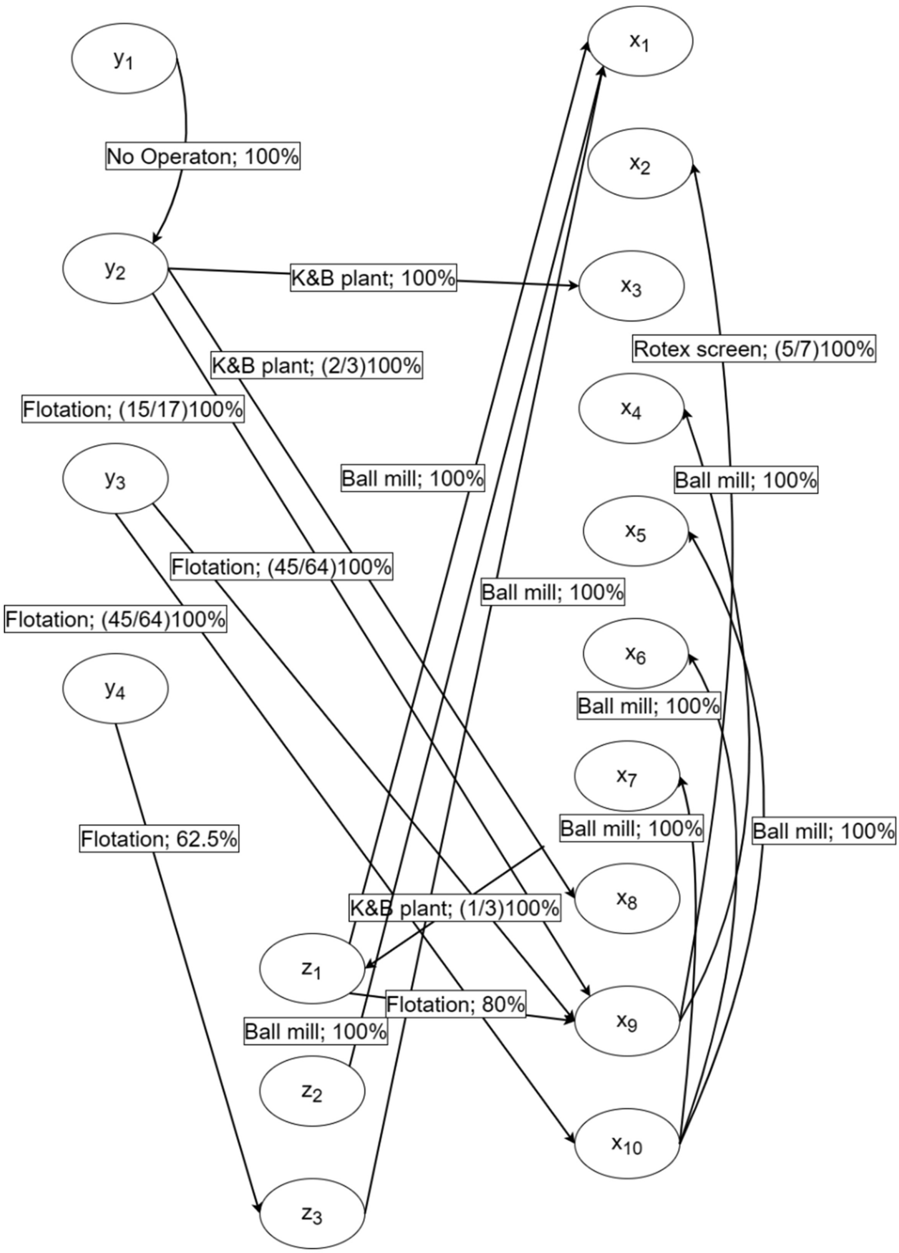
Therefore, the availability of RMs can be stated as follows:

$$A_{y_2} = a_{y_2} + P_{y_1 y_2} \quad (03)$$

Table 5. Available machine hours for each operation per month

Plant used	Available machine hrs/month
Ball mill	T_1
Rotex Screen	T_2
K & B Plant	T_3
Flotation	T_4

Source(s): Authors' own work



Source(s): Authors' own work

Figure 3.
Overall production network of the company

$$A_{y_2} = a_{y_2} \tag{04}$$

$$A_{y_2} = a_{y_2} \tag{05}$$

The Lump 97–99 RM used to produce other byproducts/products is less than the availability of Lump 97–99 (A_{y_2}) plus the conversion of Lump 99+, i.e.

$$P_{y_2,x_3} + P_{y_2,x_8} + P_{y_2,x_9} \leq A_{y_2} \tag{06}$$

The Lump 90–97 RM used to produce other byproducts/products is less than the availability of Lump 90–97 (A_{y_3}), i.e.

$$P_{y_3,x_9} + P_{y_3,x_{10}} \leq A_{y_3} \tag{07}$$

The tub dust 70–90 RM used to produce other byproducts/products is less than the availability of tub dust 70–90 (A_{y_4}), i.e.

$$P_{y_4,z_3} \leq A_{y_4} \tag{08}$$

Using the production process in the K&B plant, Lump 97–99 can be used to produce products 4676 and 9799. Therefore, the same quantity of Lump 97–99 is used to produce 4676 and 9799 dust, i.e.

$$P_{y_2,x_8} = P_{y_2,z_1} \tag{09}$$

The produced 9799 dust (using Lump 97–99 RMs) can produce F11 using the ball mill operation and Conc.99 using the flotation operation, i.e.

$$\frac{1}{3}P_{y_2,z_1} = P_{z_1,x_1} + P_{z_1,x_9} \; (9799 \, dust \, use \, to \, produce \, F11 \, and \, conc.\, 99) \tag{10}$$

Produced Conc.99 can be used to produce the products 99 powder and 2440 using the Rotex screen. Therefore, the same quantity of Conc.99 is used to produce 99 powder and 2440, i.e.

$$P_{x_9,x_2} = P_{x_9,z_2} \; (Conc.\, 99 \, use \, to \, produce \, 2440 \, and \, 99 \, powder \, is \, a \, by - product) \tag{11}$$

Produced 99 powder using the Conc.99 can be used to produce F11 using the ball mill operation, i.e.

$$\frac{2}{7}P_{x_9,z_2} = P_{z_2,x_1} \; (99 \, powder \, use \, to \, produce \, F11) \tag{12}$$

Produced Conc.97 using Tubdust 70–90 can be used to produce F11 using the ball mill operation, i.e.

$$\frac{5}{8}P_{y_4,z_3} = P_{z_3,x_1} \; (Conc.\, 97 \, use \, to \, produce \, F11) \tag{13}$$

The total quantity of F11 produced using different RMs and byproducts should be less than the demand for F11 (D_{x_1}), i.e.

$$Q_{x_1} \leq D_{x_1} \; (F11 \, demand) \tag{14}$$

$$Q_{x_1} = P_{z_1,x_1} + P_{z_2,x_1} + P_{z_3,x_1} \; (F11 \, quantity) \tag{15}$$

The total quantity of 2440 produced using different RMs and byproducts should be less than the demand of 2440 (D_{x_2}), i.e.

$$Q_{x_2} \leq D_{x_2} \; (2440 \, demand) \tag{16}$$

$$Q_{x_2} = \frac{5}{7} P_{x_9,x_2}\ (2440\ quantity) \tag{17}$$

The total produced BC9799 quantity using different RMs and byproducts should be less than the demand for BC9799 (D_{x_3}), i.e.

$$Q_{x_3} \le D_{x_3}\ (BC9799\ demand) \tag{18}$$

$$Q_{x_3} = P_{y_2,x_3}\ (BC9799\ quantity) \tag{19}$$

The total produced BP99S quantity using different RMs and byproducts should be less than the demand for BP99S (D_{x_4}), i.e.

$$Q_{x_4} \le D_{x_4}\ (BP99S\ demand) \tag{20}$$

$$Q_{x_4} = P_{x_9,x_4}\ (BP99S\ quantity) \tag{21}$$

The total produced BP98S quantity using different RMs and byproducts should be less than the demand for BP98S (D_{x_5}), i.e.

$$Q_{x_5} \le D_{x_5}\ (BP98S\ demand) \tag{22}$$

$$Q_{x_5} = P_{x_{10},x_5}\ (BP98S\ quantity) \tag{23}$$

The total produced BCB25S quantity using different RMs and byproducts should be less than the demand for BCB25S (D_{x_6}), i.e.

$$Q_{x_6} \le D_{x_6}\ (BCB25S\ demand) \tag{24}$$

$$Q_{x_6} = P_{x_{10},x_6}\ (BCB25S\ quantity) \tag{25}$$

The total produced 8148 quantity using different RMs and byproducts should be less than the demand for 8148 (D_{x_7}), i.e.

$$Q_{x_7} \le D_{x_7}\ (8148\ demand) \tag{26}$$

$$Q_{x_7} = P_{x_{10},x_7}\ (8148\ quantity) \tag{27}$$

The total produced 4676 quantity using different RMs and byproducts should be less than the demand for 4676 (D_{x_8}), i.e.

$$Q_{x_8} \le D_{x_8}\ (4676\ demand) \tag{28}$$

$$Q_{x_8} = \frac{2}{3} P_{y_2,x_8}\ (4676\ quantity) \tag{29}$$

The quantity of different RMs and byproducts used to produce Conc.99 should equal the optimum Conc.99 quantity and the quantity used to produce other products using Conc.99, i.e.

$$\frac{15}{17} P_{y_2,x_9} + \frac{45}{64} P_{y_3,x_9} + \frac{4}{5} P_{z_1,x_9} = P_{x_9,z_2} + P_{x_9,x_4} + Q_{x_9}\ (Conc.\ 99\ usage) \tag{30}$$

The Conc.99 quantity using different RMs and byproducts should be less than the demand for Conc.99 (D_{x_9}), i.e.

$$Q_{x_9} \le D_{x_9}\ (Conc.\ 99\ demand) \tag{31}$$

The quantity of different RMs and byproducts used to produce Conc.98 should equal the optimum Conc.98 quantity and the quantity used to produce other products using Conc.98, i.e.

$$\frac{45}{64}P_{y_3,x_{10}} = P_{x_{10},x_5} + P_{x_{10},x_6} + P_{x_{10},x_7} + Q_{x_{10}}\ (Conc.\,98\,usage) \tag{32}$$

The Conc.98 quantity using different RMs and byproducts should be less than the demand for Conc.98 ($D_{x_{10}}$), i.e.

$$Q_{x_{10}} \le D_{x_{10}}\ (Conc.\,98\,demand) \tag{33}$$

The total machine operation time should be less than the time availability at the K&B plant (T_1), i.e.

$$t_{X_3}P_{y_2,x_3} + t_{X_8}P_{y_2,x_8} \le T_1\ (Time\ availability\ at\ the\ K\&B\ plant) \tag{34}$$

Total machine operation time should be less than the time availability in Flotation operations (T_2), i.e.

$$t_{X_9}P_{y_2,x_9} + t_{X_9}P_{z_1,x_9} + t_{X_9}P_{y_3,x_9} + t_{Z_3}P_{y_4,z_3} \le T_2\ (Time\ availability\ at\ Flotation) \tag{35}$$

Total machine operation time should be less than the time availability at the Ball mill (T_3), i.e.

$$\begin{aligned} & t_{X_1}P_{z_1,x_1} + t_{X_1}P_{z_2,x_1} + t_{X_1}P_{z_3,x_1} + t_{X_4}P_{x_9,x_4} + t_{X_5}P_{x_{10},x_5} + t_{X_6}P_{x_{10},x_6} \\ & + t_{X_7}P_{x_{10},x_7} \le T_3\ (Time\ availability\ at\ the\ Ball\ mill) \end{aligned} \tag{36}$$

Total machine operation time should be less than the time availability at the Rotex screen (T_4), i.e.

$$t_{X_2}P_{x_9,x_2} \le T_4\ (Time\ availability\ at\ the\ Rotex\ screen) \tag{37}$$

There is a minimum production requirement (M) to maintain a buffer stock of products in the company for urgent needs, i.e.

$$Q_k \ge M_k \tag{38}$$

All the decision variables should be greater than or equal to zero, i.e.

$$P_{ij}, P_{ik}, P_{jk}, P_{kj}, P_{kk}, P_{hi}, Q_k \ge 0\ (Non-negativity\ condition) \tag{39}$$

5. Analysis and results

The formulated LP model in section 4 can be used to determine the optimal graphite mix and analysis by using the following data in Tables 6–8 related to the graphite mining process, such as weight quantities, costs, profits, operations and production processes in a selected month, as a demonstration. Table 6 shows the product names with the mathematical notations used for model formulation, actual demand in the global market, quantities needed for one full container load to satisfy customers (1FCL basis), cost and revenue for each SP, and average time requirement to produce one metric ton. In addition, Table 7 provides information regarding the carbon percentage with the form, mathematical notations and the mined amount in metric tons in the selected month. Table 8 provides the available machine hours in each machine/production plant.

The Microsoft Excel Solver add-in was used to optimize the LP model, where the solver used the simplex method to obtain the optimum solution. According to the optimal solution,

Product name	Mathematical notation of the product	Demand in Euros/Month	1FCL basis (MT)	Cost in Euros/MT	Revenue in Euros/MT	Machine hours required/MT
F 11	X_1	100	22	1994	3323	2 ½
2440	X_2	20	20	2025	3266	1 ½
BC 9799	X_3	50	22	1333	2050	¾
BP 99 S	X_4	80	22	1270	1867	3
BP 98 S	X_5	60	22	1188	1747	2 ½
BCB 25	X_6	80	22	1208	1725	2 ½
8148	X_7	40	22	1190	1700	2 ½
4676	X_8	70	22	1066	1480	1 ½
Conc. 99	X_9	60	22	1017	1356	1 ¾
Conc. 98	X_{10}	45	22	800	1067	1 ½
Powder	Z_1	–		–	–	1 ½
Powder	Z_2	–		–	–	1 ½
Powder	Z_3	–		–	–	1 ½

Source(s): Authors' own work

Table 6. Product information in the selected month

Notation of the raw material	Form of the RM	Carbon percentage	Mined quantity/month
Y_1	Lumps	99+	4
Y_2	Lumps	97–99	84
Y_3	Lumps/chips	90–97	64
Y_4	Tub dust	70–90	192
Y_5	Tub dust	Below 70	–
Y_6	Pure rock	0	–

Source(s): Authors' own work

Table 7. Raw material information in the selected month

Plant used	Available machine hrs/month
Ball mill	350
Rotex Screen	350
K & B Plant	350
Flotation	350

Source(s): Authors' own work

Table 8. Available machine hours for each operation in the selected month

the company should not produce BCB25, 8148, Conc.99, and Conc.98, and the maximum total profit is €221077.03. In addition, since the actual profit of the company for the considered month is €140298, it is clear that the formulated mathematical model is extremely beneficial to the company as far as managerial decisions are concerned. On the other hand, the advantage of the developed model was that it could identify the RM quantities allocated to each production process and to find the byproduct quantities produced and byproduct quantities used to manufacture the SPs. Therefore, this model can trace the production process to reveal the optimum managerial decisions. For example, the optimum Lump 97–99 quantity used to produce 4676 and the byproduct 9799 dust using the K&B plant is 25.85 metric tons (constraint 09). Produced Conc.97 using Tubdust 70–90 can be used to produce F11 using the ball mill operation. The optimum quantity of produced Conc.97 using Tubdust 70–90 to produce F11 using the ball mill operation is 92 metric tons (constraint 13).

The economic gain of the whole production process with the use of the LP model in one particular month, data in Table 9, shows the economic gain achieved by applying the LP model in production against (as assumed) the fact that all product categories were produced for one full container load (i.e. 22MT except for 2440 grade, 20MT of 2440 as minimum demand in the considered month) to satisfy customers by supplying at least one full container load (1FCL). The profit or economic gain comparison between one full container load of each category and production using the LP model showed sizeable differences. From this point of view, it is clear that the graphite company should use quantitative research methods of LP to determine their optimal product mix. This leads to the following results:

(1) The company's profit can be improved (from €140298 to €221077 in that particular month).

(2) According to the optimum solutions, the company should produce six SPs. However, the profit can be improved by 57.6% (from €140298 to €221077)

5.1 Sensitivity analysis

Finding the optimal solution to a LP model is essential but is not the only information available. It would also be beneficial to determine the impact on net profit when there is a change in price or cost. Sensitivity analysis (SA) studies reveal how uncertainty in a model output can distribute among different sources of uncertainty in the model input. Therefore, sensitivity analysis is diagnostic or prognostic and is considered a prerequisite for model building in any setting in any field. In this case study, sensitivity analysis plays a crucial part because of the following expected gains and disruptions.

The disruptions are,

(1) The quantity of graphite extracted cannot be guaranteed, as there are many unforeseen obstacles or reasons during underground production (mining). These are poor vein widths, collapses of underground workplaces, and vital machine breakdowns such as main hoists, underground pumps, underground communication systems, compressors that supply compressed air to underground drilling, and pumping needs. Therefore, such situations adversely affect the monthly expected graphite production required to fulfill the demand.

Table 9. Comparison between supplying 1FCL of each product category and the LP model

Product name	Profit in Euros/MT	Production in MT: 1FCL basis	Production in MT: According to the LP model	Total profit in Euros: 1FCL basis	Total profit in Euros: According to the LP model
F 11	1,329	22	100.0	29,238	132,900
2440	1,241	20	20.0	24,820	24,820
BC 9799	717	22	50.0	15,774	35,850
BP 99 S	597	22	26.92	13,134	16073.27
BP 98 S	559	22	7.69	12,298	4299.78
BCB 25	517	22	0	11,374	0
8148	510	22	0	11,220	0
4676	414	22	17.23	9,108	7133.97
Conc. 99	339	22	0	7,458	0
Conc. 98	267	22	0	5,874	0
Total		218	221.85	140,298	221077.03

Source(s): Authors' own work

(2) The uniqueness of most mining and processing machinery makes it arduous to procure spare parts and services, and a monopoly exists among the manufacturing and supplying companies involved. Most of the time, no local agents are available to provide spare parts and services for such mills and machinery.

(3) Bad climatic conditions, especially during high rainfall seasons, and an accumulation of a huge quantity of water in underground working places hinder smooth graphite extraction activities, causing less monthly underground mine production.

(4) The lack of skilled miners, especially in underground mining, is another grave issue, and training for such skills is time-consuming.

(5) Specific strict regulations limit the use of miners for overtime work, resulting in difficulties in achieving monthly production targets.

The following analysis can be done using the generated sensitivity report of the solved LP model.

If there is no minimum production requirement, the optimal solution for some variables can be taken as zero. For example, the optimal solution of the variables for this maximization problem is $P_{x_1} = 100, P_{x_2} = 20, P_{x_3} = 50, P_{x_4} = 26.92, P_{x_5} = 7.69, P_{x_6} = 0, P_{x_7} = 0,\ P_{x_8} = 17.23, P_{x_9} = 0, P_{x_{10}} = 0$ and so on. This implies that variable $P_{x_6}, P_{x_7}, P_{x_9}, and\, P_{x_{10}}$ are not profitable enough (in the maximization problem), so they stay zero. That is, the company should not produce BCB25, 8148, Conc. 98 and Conc. 99. However, if the company does not produce those, there will be a shortage of some products manufactured using those products in the market.

The reduced cost is the amount that the objective coefficient of the variable would have to be changed by before it would become profitable or cost-efficient to give the variable a positive value in the optimal solution. For example, according to the sensitivity report, the variable P_{x_9} has a reduced cost of -120.80; the objective coefficient of that variable would have to be decreased by 120.80 units in a maximization problem, and/or increased by 120.80 units in a minimization problem for the variable to become an attractive alternative to enter into the solution.

Also, the reduced cost of a given decision variable can be interpreted as the value of the objective function will deteriorate by for each unit change in the optimized value of the decision variable, with all other data held fixed. For example, according to the sensitivity report, the reduced cost of the variable P_{x_9} is -120.80. If the company decides to produce one unit (metric tons) of P_{x_9} (Conc. 99), the maximum profit will reduce by €120.80. In addition, according to the sensitivity report, the objective coefficients of each decision variable of the objective function can be changed to the allowable increased and decreased values without changing the optimal solution.

Furthermore, if an objective coefficient of a variable changes between the allowable values, then the optimal solution will not change, and it would be possible to calculate the optimal objective function value (i.e. total profit). However, if there are simultaneous changes, the 100% rule should be used, to check whether the optimum solution will change or not. If it is less than 100%, the optimal solution will remain optimal, and the total profit will change.

$$\text{i.e.} \sum \frac{Proposed\ change}{Allowable\ change} = 100\%$$

On the other hand, the shadow price tells us how much the objective value will change if a constraint's right-hand side (RHS) is increased or decreased by 1 unit. If a constraint is binding (zero slack, as a sign of limited resources in the RHS), the shadow prices of that constraint (in the maximization problem) will be positive. The shadow price denotes the economic value of the constraint's right-hand side. For example, according to the sensitivity report, the shadow price of the constraint of Lumps 99+ availability is 395.07. Suppose the availability of Lumps 99+ (RHS value) increases or decreases by one unit, which is between the allowable increase and decrease values, then, the optimum objective function value will increase or decrease by €395.07.

5.2 Scenario analysis

Since the graphite extraction quantity cannot always be guaranteed because of many unforeseen obstacles, such as poor vein widths, collapses of underground workplaces, and vital machine breakdowns, the underground mining process should be considered uncertain. Therefore, such situations affect the monthly expected graphite production requirement to fulfill sales demands. For example, since the company could not mine the expected amount of RMs because of the collapse of the underground, suppose the company produced only 1, 45, 56, and 230 metric tons of Lump 99+, Lump 97–99, Lump 90–97, and Tub dust 70–90, respectively. Since these amount variations exceed the allowable decrease values of the sensitivity report (Lump 97–99, Lump 90–97), this problem needs to be solved again. Therefore, the optimum objective function value of the new model is €197,492.88.

Besides, due to the uniqueness of most mining and processing machinery, it is very difficult to procure spare parts, and that would affect the production process because if there were some machine breakdowns, the available machine hours would reduce. For example, suppose the available machine hours were reduced by 100 since there was a ball mill failure. Since this is out of range of the allowable decrease value of the time availability at the Ball Mill constraint in the sensitivity report, this problem needs to be solved again. Therefore, the optimum objective function value of the new model is €213,112.52.

Overall, mineral manufacturing companies have to be aware of the challenges and uncertainties involved in the mining and processing of graphite, such as various unforeseen obstacles, machine breakdowns, and emergencies when making important managerial decisions.

6. Discussion

Natural graphite has a high global demand for many industrial applications (Olson *et al.*, 2016). However, only a few countries are capable of meeting this demand. The European Union and the USA have declared that graphite is a critical mineral supply because of the depletion of RMs (Olivetti *et al.*, 2017). Therefore, the derivation of a model to find the optimum graphite product mix for the mineral mixing problem to maximize the total profit according to the availability of RMs, machine capacities, etc. would bolster the effort to gain a high income according to the high global demand and to facilitate managerial decision making in graphite mining companies. Similar studies have been conducted for the mineral mixing problem in the past, and recently a study by Chanda (2018) was carried out to optimize the production planning of a mining and metallurgical complex via a network LP formulation. However, those studies failed to address the byproduct information and to analyze the production process with sensitivity and scenario analyses for further managerial decisions. Thus, this paper developed a general network LP model to optimize the product mix for the mineral mixing problem in graphite mining production and conducted a case study to demonstrate the model to address the research gaps. The research gap includes the optimization techniques used for the graphite mining production process with sensitivity and scenario analyses that were not easily accessible, and there was a lack of Network LP models to analyze production distribution systems in the mining industry. The case study also compared the economic gain achieved by applying the LP model in production against all product categories produced for one full container load to find the importance of the model formulation similar to the study conducted by Rajak *et al.* (2022). Some additionally important managerial decisions should consider can be listed as follows:

(1) How to increase the foreign currency reserves by maximizing profit and increasing government income

(2) How to improve the available manufacturing mechanisms and machinery

(3) How to improve the facilities and other benefits available for employees

(4) How to manage the available workforce, further recruitment decisions, and provide an indirect income to the population living close to the manufacturing site

Furthermore, according to the case study, it is clear that the general model for the mineral mixing problem can be customized according to different requirements and constraints to find the optimal solution. The model formulation, according to the network model, reduces the complexity of the formulation process, which provides a better understanding of the whole production process in detail, and similar research procedures were done recently for supply chain analysis by Kazancoglu *et al.* (2022) and Foroozesh *et al.* (2022). It means that the network model provides a more comprehensive view of the production process, allowing for a deeper understanding of how different components of the process are interconnected and interact with each other, of which the case study of this research provides a useful example. Another key fact of this model regarding the production process is that the SPs that can be produced by using the SPs and byproducts that can be produced by using the byproducts/SPs can be identified where Göthe-Lundgren (2002), Khan *et al.* (2018), and Fischer *et al.* (2004) also addressed similar optimization techniques that can be used to find the same information. In addition, the formulated LP models can be easily solved by the Simplex Method, which is included in the Microsoft Excel solver add-in and can generate sensitivity analysis and answer reports to initiate further managerial decisions based on the optimal solutions and optimality ranges where Yadav *et al.* (2022) has done a similar research work to address the sensitivity analysis by considering the byproducts and sustainable manufacturing.

However, this study has some limitations. The mineral manufacturing industry should highly consider the uncertainties involved with mining and processing, such as machine breakdowns and emergencies. Therefore, it would advocate making efficient managerial decisions if the mineral mixing process was taken under uncertainty and use stochastic optimization techniques to find the optimum production schedule. Hence, this study can be extended, such as the studies conducted by Noriega *et al.* (2022), Dimitrakopoulos (2011), and Navarra *et al.* (2018).

7. Conclusion

This research aimed to find the optimal use of graphite RMs in various graphite grades to maximize the total profit based on customer requirements and available resources for the mineral mixing problem because of the sparsity of optimization techniques used for the graphite mining production process. This is done to find the details of SPs, byproducts, and byproduct operations. The objective is to analyze the production process to maximize the profit with sensitivity and scenario analyses. Besides, a network model bolsters the transformation of a production process into a mathematical model without implementing it at once when the process is complex and challenging. This research exemplified that the overall production process could be visualized as a network to identify the complexity of the process, flow, and the bottleneck of the production process, since the products and byproducts of the RMs with different production processes, varying times of production, and different carbon percentages make a complex problem. Therefore, the overall production process was mapped as a network and implemented as a general LP model to find the optimum product mix for the mineral mixing problem. In addition, a case study tested the proposed model on a graphite mining company that mines approximately 400 metric tons of raw graphite per month, which is sorted into six raw graphite types to produce ten types of graphite grades. According to the optimum solution, the company should produce only six of these ten products to maximize its total profit, where the case study ascertained that the generation of answer and sensitivity reports for an LP model could be used to compile the sensitivity analysis for the different parameter changes, and scenario analysis was advocated since the researchers realized the challenges and uncertainties involved in the mining and processing of graphite, which will impact important managerial decisions. Furthermore, the case study concluded that a generated LP model via a network model to find the optimal product mix for the mineral mixing problem derived from the available data is the most suitable tool to obtain the maximum profit and to facilitate consequential managerial decisions in the graphite manufacturing industry. The study concluded with a case study which implies that the developed general LP model with the theoretical contribution can be used to optimize the graphite mining production process by customizing the model according to the requirements and constraints. In addition, the model was used to identify the information on SPs that can be produced by using the SPs and byproducts that can be produced by using the byproducts/SPs.

Besides, the underground mining process should be considered uncertain since the graphite extraction quantity cannot always be guaranteed due to many unforeseen obstacles. Moreover, the machine breakdown times also should consider uncertain because the available machine hours could be reduced. Therefore, it would be beneficial to undertake further research on several topics. These include the production cost variation, mining technique changes, mining quantity variations, production time variations, bottleneck identification, and an estimation of how realistic the production time/available mining quantity/production cost under uncertainty is by comparing the actual data.

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