Guest editorial

On the topological derivative method and its applications in computational engineering

The topological derivative (or topological gradient) concept has been designed in order to give a precise and quantitative information on the sensitivity of a user-given shape functional with respect to topological domain perturbations. Specifically, it appears in the first term of the asymptotic expansion of the shape functional with respect to a small parameter measuring the size of the perturbation under consideration, typically a hole, an inclusion, a source-term or a crack. It naturally complements the more classical notion of shape derivative that accounts for smooth shape variations. As a matter of fact, the two concepts can be explicitly related through the expression of the topological derivative as the singular limit of the shape gradient (Novotny and Sokołowski, 2013).

The origin of the topological derivative method in optimal design can be dated to the work of Eschenauer *et al.* (1994) on the optimal location of holes within elastic structures. It is nevertheless worth mentioning prior related mathematical developments, on the one hand, on the asymptotic behavior of solutions to singularly perturbed boundary value problems, and on the other hand, on the classical notions of polarization and capacity matrices. These latter objects enter as essential building blocks in the formulation of topological derivatives. The first mathematical justifications for topological derivatives in the framework of partial differential equations are due to Sokołowski and Żochowski (1999) and Garreau *et al.* (2001), in the context of the Poisson equation and the Navier system for Neumann and Dirichlet holes.

In the last decade, the topological sensitivity analysis has become a rich and fascinating research field that combines the modern theory of calculus of variations, partial differential equations, differential geometry, numerical analysis, physics, engineering and computational mechanics. The field grew up rapidly to develop many extensions and address a variety of physical and industrial problems. Without being exhaustive, the topological derivative method finds its main applications in shape and topology optimization, geometrical inverse problems, image processing, multi-scale material design and mechanical modeling, including damage and fracture evolution phenomena.

The research challenges for the years to come include mathematical aspects in nonlinear problems, such as geometrical, constitutive and kinematic nonlinearities, as well as their applications in multi-physic and multi-objective topology optimization of large structures, for instance. The optimal design of complex topologies at different scales, possibly manufactured by 3D printing or other emerging technologies, such as micro-architectured materials and nanophotonic devices, is a research field to be considered with a promising future. Finally, the development of onboard codes for structural defects imaging in real-time is also an example of research challenge in the industrial applications of the topological derivative.

The research community related to the topological derivative encompasses applied mathematicians, computer scientists and engineers in several disciplines. This special issue contains 16 selected papers from the most active experts on the field, covering various topics ranging from new theoretical developments to applications in structural and fluid dynamics topology optimization, geometrical inverse problems, synthesis and optimal design of metamaterials, fracture mechanics modeling, up to industrial applications and experimental validation. We hope that the reader will benefit from the developments and applications presented herein and will gain new perspectives on the topic. Finally, we want to acknowledge the efforts of the authors in preparing each contribution for this issue, in which the mathematical background and scientific rigor of the selected manuscripts have to be noted.

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