An integrated approach to grey relational analysis, analytic hierarchy process and data envelopment analysis

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Abstract

Purpose – This paper aims to propose an integration of the analytic hierarchy process (AHP) and data envelopment analysis (DEA) methods in a multiattribute grey relational analysis (GRA) methodology in which the attribute weights are completely unknown and the attribute values take the form of fuzzy numbers.

Design/methodology/approach – This research has been organized to proceed along the following steps: computing the grey relational coefficients for alternatives with respect to each attribute using a fuzzy GRA methodology. Grey relational coefficients provide the required (output) data for additive DEA models; computing the priority weights of attributes using the AHP method to impose weight bounds on attribute weights in additive DEA models; computing grey relational grades using a pair of additive DEA models to assess the performance of each alternative from the optimistic and pessimistic perspectives; and combining the optimistic and pessimistic grey relational grades using a compromise grade to assess the overall performance of each alternative.

Findings – The proposed approach provides a more reasonable and encompassing measure of performance, based on which the overall ranking position of alternatives is obtained. An illustrated example of a nuclear waste dump site selection is used to highlight the usefulness of the proposed approach.

Originality/value – This research is a step forward to overcome the current shortcomings in the weighting schemes of attributes in a fuzzy multiattribute GRA methodology.

Keywords Data envelopment analysis, Grey relational analysis, Analytic hierarchy process, Multiple attribute decision making, Fuzzy numbers, Weighting

Paper type Research paper

Introduction

Grey relational analysis (GRA) is a part of the grey system theory proposed by Deng (1982), which is suitable for solving a variety of multiple attribute decision making

(MADM) problems with both crisp and fuzzy data (Goyal and Grover, 2012; Wei *et al.*, 2011; Wei, 2010; Hou, 2010; Olson and Wu, 2006). Grey sets can be considered as an extension to fuzzy sets by restricting the characteristic (or membership) function values of a set within [0,1] (Yang and John, 2012; Li *et al.*, 2008). GRA solves MADM problems by aggregating multiple attribute values, which are usually incommensurable, into a single value for each alternative (Kuo *et al.*, 2008). In the traditional GRA method, assigning equal weights to attributes for each alternative is a norm. Nevertheless, the validity of using the equal weight assumption for all the alternatives to be assessed can be questioned, as each one of these has its own characteristics and preferences. Therefore, the study on attribute weighting can be an interesting, but it is a controversial topic in the field of GRA. Fortunately, the development of modern operational research has provided us with two excellent tools, namely, analytic hierarchy process (AHP) and data envelopment analysis (DEA), which can be used to derive attribute weights for use in GRA.

AHP is a subjective data-oriented procedure, which can reflect the relative importance of a set of attributes and alternatives based on the formal expression of the decision-maker's preferences. AHP usually involves three basic functions: structuring complexities, measuring on a ratio-scale and synthesizing (Saaty, 1987). Some researchers incorporate fuzzy set theory in the conventional AHP to express the uncertain comparison judgments as fuzzy numbers (Osman *et al.*, 2013; Javanbarg *et al.*, 2012; Kahraman *et al.*, 2003). However, AHP has been criticized because of the arbitrary nature of the ranking process (Ahmad *et al.*, 2006; Swim, 2001; Dyer *et al.*, 1990). In fact, the AHP weights are based on the experts' personal experiences and their subjective judgments. If the selection of experts is different, then the weights obtained will be different (Liu, 2009; Liu and Chen, 2004). The application of AHP with GRA can be seen in the studies by Birgün and Güngör (2014), Jia *et al.* (2011) and Zeng *et al.* (2007).

Alternatively, DEA is an objective data-oriented approach to assess the relative performance of a group of decision-making units (DMUs) with multiple inputs and outputs (Cooper *et al.*, 2011). Traditional DEA models require crisp input and output data. However, in recent years, fuzzy set theory has been proposed for quantifying imprecise and vague data in DEA models (Hatami-Marbini *et al.*, 2013; Wen and Li, 2009; Lertworasirikul *et al.*, 2003). In the field of GRA, DEA models without explicit inputs are applied, that is the models in which only pure outputs or index data are taken into account (Liu *et al.*, 2011). The other combined GRA and DEA methodologies can be found in the literature, such as using GRA for the selection of inputs and outputs in DEA (Wang *et al.*, 2010; Bruce Ho, 2011), using GRA for ranking efficient DMUs in DEA with crisp data (Girginer *et al.*, 2015), using GRA for ranking DMUs in DEA with grey data, that is the unknown numbers, which have clear upper and lower limits (Markabi and Sabbagh, 2014) and weighting GRA using a cross-efficiency model (Markabi and Sarbijan, 2015). However, the main problem of using traditional DEA models in GRA is that several alternatives may receive a grey relational grade of 1, which means that all of these alternatives are ranked in the first position (Jun and Xiaofei, 2013; Wu and Olson, 2010). To overcome this deficiency, Zheng and Lianguang (2013) propose a super-efficiency model, based on the super-efficiency ranking method of Andersen and Petersen (1993). According to this model, these alternatives are allowed to obtain a grey relational grade greater than 1 by removing the constraint that bounds the grade of the assessed alternative. These grades are then used to rank the alternatives and thereby

eliminate some of the ties that occur in the selection of the best alternative. As shown in all the aforementioned models, each alternative has been allowed to choose its own most favorable weights to maximize its performance, they can be called optimistic DEA models.

On the other hand, a similar approach can be developed to assess the performance of each alternative under the least favorable weights, which, in fact, is pessimistic. According to this approach, each alternative is compared with the worst alternatives and is assessed by its grey relational grade as the ratio of the distance from the worst frontier. It is worth pointing out that the worst-practice frontier approach is not a new approach in the DEA literature. Conceptually, it is parallel to the worst possible efficiency concept as discussed by Wang and Luo (2006), Takamura and Tone (2003), Jahanshahloo and Afzalinejad (2006) and Liu and Chen (2009). Nevertheless, as far as we know, this approach has never been applied to the field of GRA.

It can be argued that both optimistic and pessimistic DEA approaches should be considered together to obtain attribute weights in GRA, and any approach considers that only one of them is biased (Wang et al., 2007). In both approaches, each DMU or alternative can freely choose its own system of weights to optimize its performance. However, this freedom of choosing weights is equivalent to keeping the preferences of a decision-maker out of the decision process. In fact, an alternative may be indicated as the best (worst) one by assigning zero values to the weights of some attributes and neglecting the relative priorities of these attributes in the decision-making process.

To overcome these issues, we propose the integration of AHP and DEA to obtain the attribute weights in GRA from both the optimistic and pessimistic perspectives. This can be implemented by imposing weight restrictions in DEA-based GRA models, using AHP (Pakkar, 2016a). There are two types of weight restrictions: homogeneous and non-homogeneous (Podinovski, 2004). A homogeneous weight restriction has a zero-free constant on its right-hand side (RHS), while a non-homogeneous weight restriction has a non-zero constant on its RHS. The bounds on the relative values of input and output weights (Lee et al., 2012; Liu et al., 2005; Tseng et al., 2009; Kong and Fu, 2012) and those on the virtual weights of inputs and outputs (Premachandra, 2001; Shang and Sueyoshi, 1995) are typical examples of homogeneous weight restrictions using AHP. The absolute weight bounds that directly impose bounds on input and output weights (Entani et al., 2004; Pakkar, 2016b) belong to the type of non-homogeneous weight restrictions using AHP. The DEA models discussed in this paper are all based on the additive models (Cooper et al., 1999) without explicit inputs in which output data are defined by a fuzzy GRA method. In these models, the decision makers are allowed to reflect those considerations not embodied in data. This would allow us to easily incorporate AHP weights into the multiplier form of additive models using suitable non-homogeneous weight restrictions.

Methodology
As has been mentioned earlier, a critical issue in using the GRA method is the subjectivity in assigning weights for attributes. As different weight combinations may lead to different ranking results, it is unlikely that all the decision-makers would easily reach a consensus in determining an appropriate set of weights. In addition, it may not be easy to obtain expert information for deriving the weights. Although the use of equal

weights seems to be a relatively fair choice, some decision-makers may still have different opinions on the relative importance of attributes. To avoid these issues, an integration of AHP and DEA models is given here to obtain the attribute weights in a fuzzy GRA methodology. This can be implemented through the following steps (Figure 1):

- computing the grey relational coefficients for alternatives with respect to each attribute using a fuzzy GRA method. Grey relational coefficients provide the required (output) data for additive DEA models;
- computing the priority weights of attributes using the AHP method to impose weight bounds on attribute weights in additive DEA models;
- computing grey relational grades using a pair of additive DEA models to assess the performance of each alternative from the optimistic and pessimistic perspectives; and
- combining the optimistic and pessimistic grey relational grades using a compromise grade to assess the overall performance of each alternative.

Fuzzy grey relational analysis
GRA can be applied to both crisp and fuzzy data. Here, we use it as a means to obtain a solution from fuzzy data. Let $A = \{A_1, A_2, ..., A_m\}$ be a discrete set of alternatives and $C = \{C_1, C_2, ..., C_n\}$ be a set of attributes. Let $\tilde{y}_{ij} = (y_{1ij}, y_{2ij}, y_{3ij}, y_{4ij})$ be a trapezoidal fuzzy number representing the value of attribute $C_j(j = 1, 2, ..., n)$ for alternative $A_i (i = 1, 2, ..., m)$. Using the α-cut technique, a trapezoidal fuzzy number can be transformed into an interval number as follows:

$$y_{ij} = [y_{ij}^-, y_{ij}^+] = [\alpha y_{2ij} + (1 - \alpha)y_{1ij}, \alpha y_{3ij} + (1 - \alpha) y_{4ij})] \qquad (1)$$

where $y_{ij} = [y_{ij}^-, y_{ij}^+], y_{ij}^- \leq y_{ij}^+$, is an interval number representing the value of attribute $C_j(j = 1, 2, ..., n)$ for alternative $A_i (i = 1, 2, ..., m)$. Then alternative A_i is characterized
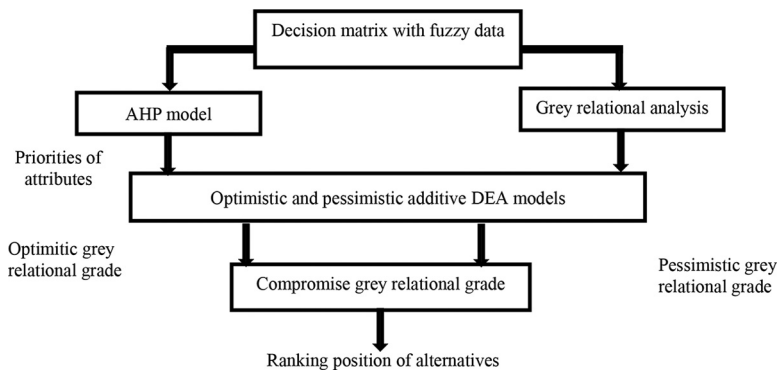

Figure 1.
An integrated approach to GRA, AHP and DEA

by a vector $Y_i = ([y_{i1}^-, y_{i1}^+], [y_{i2}^-, y_{i2}^+], \ldots, [y_{in}^-, y_{in}^+])$ of attribute values. The term Y_i can be translated into the comparability sequence $R_i = ([r_{i1}^-, r_{i1}^+], [r_{i2}^-, r_{i2}^+], \ldots, [r_{in}^-, r_{in}^+])$ by using the following equations (Zhang *et al.*, 2005):

$$[r_{ij}^-, r_{ij}^+] = \left[\frac{y_{ij}^-}{y_{j(\max)}^+}, \frac{y_{ij}^+}{y_{j(\max)}^+} \right] \forall j, \ y_{j(\max)}^+ = \max\{y_{1j}^+, y_{2j}^+, \ldots, y_{mj}^+\} \text{ for desirable attributes,}$$

(2)

$$[r_{ij}^-, r_{ij}^+] = \left[\frac{y_{i(\min)}^-}{y_{ij}^-}, \frac{y_{i(\min)}^-}{y_{ij}^+} \right] \forall j, \ y_{j(\min)}^- = \min\{y_{1j}^-, y_{2j}^-, \ldots, y_{mj}^-\} \text{ for undesirable attributes.}$$

(3)

Now, let A_0 be a virtual ideal alternative, which is characterized by a reference sequence $U_0 = ([u_{01}^-, u_{01}^+], [u_{02}^-, u_{i2}^+], \ldots, [u_{0n}^-, u_{0n}^+])$ of the maximum attribute values as follows:

$$u_{0j}^- = \max\{r_{1j}^-, r_{2j}^-, \ldots, r_{mj}^-\} \forall j,$$

(4)

$$u_{0j}^+ = \max\{r_{1j}^+, r_{2j}^+, \ldots, r_{mj}^+\} \forall j.$$

(5)

To measure the degree of similarity between $r_{ij} = [r_{ij}^-, r_{ij}^+]$ and $u_{0j} = [u_{0j}^-, u_{0j}^+]$ for each attribute, the grey relational coefficient, ξ_{ij}, can be calculated as follows:

$$\xi_{ij} = \frac{\min_i \min_j |[u_{0j}^-, u_{0j}^+] - [r_{ij}^-, r_{ij}^+]| \ + \ \rho \max_i \max_j |[u_{0j}^-, u_{0j}^+] - [r_{ij}^-, r_{ij}^+]|}{|[u_{0j}^-, u_{0j}^+] - [r_{ij}^-, r_{ij}^+]| \ + \ \rho \max_i \max_j |[u_{0j}^-, u_{0j}^+] - [r_{ij}^-, r_{ij}^+]|}$$

(6)

while the distance between $u_{0j} = [u_{0j}^-, u_{0j}^+]$ and $r_{ij} = [r_{ij}^-, r_{ij}^+]$ is measured by $|u_{0j} - r_{ij}| = \max(|u_{0j}^- - r_{ij}^-|, |u_{0j}^+ - r_{ij}^+|)$. $\rho \in [0, 1]$ is the distinguishing coefficient, which is generally $\rho = 0.5$. It should be noted that the final results of GRA for MADM problems are very robust to changes in the values of ρ. Therefore, selecting the different values of ρ would only slightly change the rank order of attributes (Kuo *et al.*, 2008). To find an aggregated measure of similarity between alternative A_i, characterized by the comparability sequence R_i, and the ideal alternative A_0, characterized by the reference sequence U_0, over all the attributes, the grey relational grade, Γ_i, can be computed as follows:

$$\Gamma_i = \sum_{j=1}^{n} w_j \xi_{ij}$$

(7)

where w_j is the weight of attribute C_j. In the next section, we show how the AHP model can be used to obtain the priority weights of attributes for each alternative.

The analytic hierarchy process
The AHP procedure for computing the priority weights of attributes may be broken down into the following steps:

Step 1: A decision-maker makes a pairwise comparison matrix of different attributes, denoted by B with the entries of b_{hq} ($h = q = 1, 2, ..., n$). The comparative importance of attributes is provided by the decision-maker using a rating scale. Saaty (1987) recommends using a 1-9 scale.

Step 2: The AHP method obtains the priority weights of attributes by computing the eigenvector of matrix B [equation (8)], $e = (e_1, e_2, ..., e_j)^T$, which is related to the largest eigenvalue, γ_{max}:

$$Be = \gamma_{max} e \tag{8}$$

To determine whether the inconsistency in a comparison matrix is reasonable the random consistency ratio, *C.R.*, can be computed by the following equation:

$$C.R. = \frac{\gamma_{max} - N}{(N - 1)R.I.} \tag{9}$$

where *R.I.* is the average random consistency index and N is the size of a comparison matrix.

Optimistic and pessimistic additive DEA models
As all the grey relational coefficients are benefit (output) data, an optimistic additive DEA model for obtaining attribute weights in GRA can be developed like the additive model in Cooper *et al.* (1999) without explicit inputs as follows:

$$
\begin{aligned}
P_k &= \max \sum_{j=1}^{n} e_j s_j^+ \\
\text{s.t. } &\sum_{i=1}^{m} \lambda_i \xi_{ij} - s_j^+ = \xi_{kj} \, \forall j, \\
&\sum_{i=1}^{m} \lambda_i = 1 \\
&s_j^+, \lambda_i \geq 0,
\end{aligned}
\tag{10}
$$

where $1 - P_k$ indicates the grey relational grade, $\Gamma_k (k = 1, 2, ..., m)$, for alternative under assessment A_k (known as a DMU in the DEA terminology) and $0 \leq P_k \leq 1$. s_j^+ is the slack variable of attribute $C_j (j = 1, 2, ..., n)$, expressing the difference between the performance of a composite alternative and the performance of the assessed alternative with respect to each attribute. In other words, s_j^+ identifies a shortfall in the attribute value of C_j for alternative A_k. Obviously, when $P_k = 0$ alternative A_k is considered as the best alternative in comparison to all the other alternatives. e_j is the priority weight of attribute C_j, which is defined out of the internal mechanism of DEA using AHP and λ_i is the weight of alternative $A_i (i = 1, 2, ..., m)$. The convexity constraint in model (10) meets the assumption of *variable returns-to-scale* (VRS)

frontier for an additive model. The dual of model (10) can be developed as follows:

$$\Gamma_k = \max \sum_{j=1}^{n} w_j \xi_{kj} - w_0$$

$$\text{s.t.} \sum_{j=1}^{n} w_j \xi_{ij} - w_0 \leq 1 \; \forall i,$$

$$w_j \geq e_j \; \forall j,$$

$$w_0 \; free.$$

(11)

This model is useful for our purpose in dealing with grey relational grades. The objective function in model (11) maximizes the ratio of the grey relational grade of alternative A_k to the maximum grey relational grade across all alternatives for the same set of weights ($\max \Gamma_k / \max_{i=1,\dots,m} \Gamma_i$), while the priority weights obtained by AHP impose the lower bounds on the attribute weights. Hence, an optimal set of weights in model (11) represents A_k in the best light compared to all the other alternatives while it reflects a *priori* information about the priorities of attributes, simultaneously. Finally, one should notice that the optimistic additive DEA models bounded by AHP does not necessarily yield results that are different from those obtained from the original additive DEA models (Charnes *et al.*, 1985). In particular, it does not increase the power of discrimination between the considerable number of alternatives, which are usually ranked in the first place by obtaining the grey relational grades of 1. To overcome these issues, we develop the additive models from the pessimistic point of view in which each alternative is assessed based on its distance from the worst practice frontier as follows:

$$P_k^{'} = \max \sum_{j=1}^{n} e_j s_j^-$$

$$\text{s.t.} \sum_{i=1}^{m} \lambda_i^{'} \xi_{ij} + s_j^- = \xi_{kj} \; \forall j,$$

$$\sum_{i=1}^{m} \lambda_i^{'} = 1$$

$$s_j^-, \lambda_i^{'} \geq 0,$$

(12)

Note that the only difference between model (10) and model (12) is the signs of slack variables in the first set of constraint. s_j^- is the slack variable of attribute $C_j (j = 1, 2, \dots, n)$, expressing the difference between the performance of the assessed alternative and the performance of a composite alternative with respect to each attribute. In other words, s_j^- identifies the excess values of attribute C_j for alternative A_k. This is obvious when $P_k^{'} = 0$ alternative A_k is considered as the worst alternative compared to all the other alternatives. In some cases, the worst alternatives, however, may also be the best alternatives. This happens when the assessed alternative is the best in some attributes, while it is the worst in some other attributes. The dual of model (12) can be developed as follows:

$$\Gamma'_k = \min \sum_{j=1}^{n} w'_j \xi_{kj} + w'_0$$

$$\text{s.t. } \sum_{j=1}^{n} w'_j \xi_{ij} + w'_0 \geq 1 \ \forall i, \tag{13}$$

$$w'_j \geq e_j \ \forall j,$$

$$w'_0 \text{ free.}$$

Here, we seek the worst weights in the sense that the objective function in model (13) is minimized. The first set of constraints assures that the computed weights do not attain a grade smaller than 1. Each alternative is compared with these worst alternatives and is assessed based on the ratio of the distance from the worst-practice frontier. It is worth pointing out that the pessimistic additive models in this paper are not brand-new models in the DEA literature. Conceptually, it is parallel to the additive DEA models as discussed by Jahanshahloo and Afzalinejad (2006) for ranking alternatives on a full inefficient-frontier. Nevertheless, as far as we know, it is the first time that they are applied to the field of GRA.

To combine the grey relational grades obtained from models (11) and (13), that is the best and worst sets of weights, the linear combination of corresponding normalized grades is recommended as follows (Zhou et al., 2007):

$$\Delta_k(\beta) = \beta \frac{\Gamma_k - \Gamma_{\min}}{\Gamma_{\max} - \Gamma_{\min}} + (1 - \beta) \frac{\Gamma'_k - \Gamma'_{\min}}{\Gamma'_{\max} - \Gamma'_{\min}} \tag{14}$$

where $\Gamma_{\max} = \max \{\Gamma_k, k = 1, 2, ..., m\}$, $\Gamma_{\min} = \min \{\Gamma_k, k = 1, 2, ..., m\}$, $\Gamma'_{\max} = \max \{\Gamma'_k, k = 1, 2, ..., m\}$, $\Gamma'_{\min} = \min \{\Gamma'_k, k = 1, 2, ..., m\}$ and $0 \leq \beta \leq 1$ is an adjusting parameter, which may reflect the preference of a decision-maker on the best and worst sets of weights. $\Delta_k(\beta)$ is a normalized compromise grade in the range [0,1].

Numerical example: nuclear waste dump site selection
In this section, we present the application of the proposed approach for nuclear waste dump site selection. The multi-attribute data, adopted from Wu and Olson (2010), are presented in Table I. There are 12 alternative sites and 4 performance attributes. Cost, lives lost and risk are undesirable attributes and civic improvement is a desirable attribute. Cost is in billions of dollars. Lives lost reflect expected lives lost from all exposures. Risk shows the risk of catastrophe (earthquake, flood, etc.) and civic improvement is the improvement of the local community due to the construction and operation of each site. Cost and lives lost are crisp values as outlined in Table I, but risk and civic improvement have fuzzy data for each nuclear dump site.

We use the processed data as reported by Wu and Olson (2010). First the trapezoidal fuzzy data are used to express linguistic data in Table I. Using the α-cut technique, the raw data are expressed in fuzzy intervals as shown in Table II. These data are turned into the comparability sequence by using equations (2) and (3). Each attribute is now on a common 0-1 scale where 0 represents the worst imaginable attainment on an attribute, and 1 represents the best possible attainment.

Table III shows the results of a pairwise comparison matrix in the AHP model as constructed by the author in Expert Choice software. The priority weight for each attribute would be the average of the elements in the corresponding row of the normalized matrix of pairwise comparison, shown in the last column of Table III. One can argue that the priority weights of attributes must be judged by nuclear safety experts. However, as the aim of this section is just to show the application of the proposed approach on numerical data, we see no problem to use our judgment alone.

Site	Cost	Lives	Risk	Civic
Nome	40	60	Very high	Low
Newark	100	140	Very low	Very high
Rock Springs	60	40	Low	High
Duquesne	60	40	Medium	Medium
Gary	70	80	Low	Very high
Yakima	70	80	High	Medium
Turkey	60	70	High	High
Wells	50	30	Medium	Medium
Anaheim	90	130	Very high	Very low
Epcot	80	120	Very low	Very low
Duckwater	80	70	Medium	Low
Santa Cruz	90	100	Very high	Very low

Table I.
Data for nuclear waste dump site selection

Site	Cost	Lives lost	Risk	Civic
Nome	[0.80-1.00]	[0.40-0.70]	[0.00-0.10]	[0.10-0.30]
Newark	[0.00-0.05]	[0.00-0.05]	[0.90-1.00]	[0.90-1.00]
Rock Springs	[0.70-0.95]	[0.70-0.90]	[0.70-0.90]	[0.70-0.90]
Duquesne	[0.50-0.85]	[0.70-0.90]	[0.40-0.60]	[0.40-0.60]
Gary	[0.40-0.60]	[0.10-0.30]	[0.70-0.90]	[0.90-1.00]
Yakima	[0.50-0.70]	[0.10-0.30]	[0.10-0.30]	[0.40-0.60]
Turkey	[0.75-0.90]	[0.20-0.40]	[0.10-0.30]	[0.70-0.90]
Wells	[0.85-0.95]	[0.85-1.00]	[0.40-0.60]	[0.40-0.60]
Anaheim	[0.00-0.30]	[0.00-0.10]	[0.00-0.10]	[0.00-0.10]
Epcot	[0.10-0.40]	[0.00-0.20]	[0.90-1.00]	[0.00-0.10]
Duckwater	[0.30-0.50]	[0.20-0.40]	[0.40-0.60]	[0.10-0.30]
Santa Cruz	[0.10-0.40]	[0.10-0.30]	[0.00-0.10]	[0.00-0.10]

Table II.
Fuzzy interval nuclear waste dump site data

Attribute	Cost	Lives	Risk	Civic	Priority
Cost	1	1/5	1/2	3	0.131
Lives	5	1	2	9	0.545
Risk	2	1/2	1	6	0.275
Civic	1/3	1/9	1/6	1	0.05

Note: C.R. = 0.01

Table III.
Pairwise comparison matrix of four attributes

Using equation (6), all grey relational coefficients are computed to provide the required (output) data for additive DEA models as shown in Table IV. Note that the grey relational coefficients depend on the distinguishing coefficient ρ, which here is 0.80.

Table V presents the results obtained from models (11) and (13), as well as the corresponding composite grades at $\beta = 0.5$. If decision-makers have no strong preference, $\beta = 0.5$ would be a fairly neutral and reasonable choice. It can be seen from Table V, the Wells site, with a compromised grade of 1, stands in the first place, while six other alternatives are ranked in the first position by model (11). It is likely because of the fact that the Wells site not only has relatively high values of grey relational coefficients but also has a better combination among the different attributes. This indicates that the proposed approach can significantly improve the degree of discrimination among alternatives. We can also observe that Newark is the best alternative from the optimistic point of view, but it is also the worst alternative from the pessimistic point of view. It is due to the fact that Newark is the best with respect to risk and civic improvement, while it is the worst with respect to

Site	Cost	Lives lost	Risk	Civic
Nome	0.9383	0.6281	0.4578	0.4872
Newark	0.4444	0.4444	1	1
Rock Springs	0.8352	0.8352	0.7917	0.7917
Duquesne	0.6847	0.8352	0.6032	0.6032
Gary	0.6281	0.5033	0.7917	1
Yakima	0.6847	0.5033	0.4872	0.6032
Turkey	0.8837	0.539	0.4872	0.7917
Wells	0.9383	1	0.6032	0.6032
Anaheim	0.472	0.4578	0.4578	0.4578
Epcot	0.5033	0.472	1	0.4578
Duckwater	0.5802	0.539	0.6032	0.4872
Santa Cruz	0.5033	0.5033	0.4578	0.4578

Table IV.
Results of grey relational coefficients for nuclear waste dump site selection

Site	Γ_k	Γ'_k	$\Delta_k(\beta = 0.5)$
Nome	0.7515	1.1554	0.3848 (8)
Newark	1.0000	1.0000	0.5000 (7)
Rock Springs	1.0000	1.3618	0.9480 (2)
Duquesne	0.8770	1.2808	0.6954 (5)
Gary	1.0000	1.1642	0.7033 (3)
Yakima	0.6642	1.068	0.1684 (10)
Turkey	1.0000	1.123	0.6523 (6)
Wells	1.0000	1.4038	1.0000 (1)
Anaheim	0.5962	1.0000	0.0000 (12)
Epcot	1.0000	1.1609	0.6992 (4)
Duckwater	0.6960	1.0999	0.2474 (9)
Santa Cruz	0.6251	1.0289	0.0716 (11)

Table V.
Results of grey relational grades obtained from models (11) and (13) and the corresponding compromise grades[a]

Note: [a] The site ranks are given in parentheses

cost and lives lost. Therefore, one of the advantages of the proposed approach is revealing such alternatives.

Comparing proposed model and Wu-Olson model

The model of Wu and Olson is similar to the CCR (after Charnes *et al.* 1978) model (after Charnes, Cooper, & Rhodes, 1978) without explicit inputs. Their model can be obtained by setting $e_j = 0$ and $w_0 = 0$ in model (11). By setting $e_j = 0$, each alternative is allowed to choose its own most favorable weights without using a *priori* weighting and by setting $w_0 = 0$, the assumption of *constant-returns-to-scale* (CRS) in the CCR models is satisfied. Nonetheless, if an alternative has the largest grey relational coefficient in comparison to the other alternatives for a certain attribute, this alternative would always obtain a grey relational grade of 1, even if it has an extremely small grey relational coefficient for other attributes (see Appendix for the mathematical proof). This may lead to the situation in which a large number of alternatives are ranked in the first position. To avoid this issue, we propose the corresponding pessimistic formation by setting $e_j = 0$ and $w'_0 = 0$ in model (13) in which each alternative is allowed to choose its own least favorable weight without using a *priori* weighting under CRS. Under these assumptions, the results obtained from models (11) and (13), as well as the corresponding composite grades for nuclear waste dump site selection, are presented in Table VI.

Table VI shows that Rock Springs , with a compromise grade of 1, stands in the first place, while seven other alternatives are ranked in the first position by model (11). This indicates that the proposed approach can significantly improve the degree of discrimination among alternatives. It is worth noting that, although Rock Springs has the highest compromise grade (=1), it does not have the highest grey relational coefficient with respect to each attribute (Table IV). It is likely due to the fact that Rock Springs not only has relatively high values of grey relational coefficients but also has a better combination among the different attributes.

Site	Γ_k	Γ'_k	$\Delta_k(\beta = 0.5)$
Nome	1.0000	1.0000	0.5000 (6)
Newark	1.0000	1.0000	0.5000 (6)
Rock Springs	1.0000	1.7294	1.0000 (1)
Duquesne	0.8921	1.3176	0.5912 (3)
Gary	1.0000	1.1146	0.5785 (4)
Yakima	0.7855	1.0642	0.2926 (7)
Turkey	1.0000	1.0642	0.5440 (5)
Wells	1.0000	1.3176	0.7177 (2)
Anaheim	0.5735	1.0000	0.0000 (10)
Epcot	1.0000	1.0000	0.5000 (6)
Duckwater	0.7351	1.0642	0.2335 (8)
Santa Cruz	0.5943	1.0000	0.0244 (9)

Note: [a] The site ranks are given in parentheses

Table VI.
Results of grey relational grades obtained from models (11) and (13) and the corresponding compromise grades without using a *priori* weighting under CRS[a]

Conclusions

In this paper, we present the integration of AHP and DEA models in a fuzzy GRA methodology to obtain the weights of attributes. The two sets of grey relational grades obtained from the most and least favorable weights measure the two extreme performances of each alternative. However, any assessment approach considering only one of them is biased. The compromise grey relational grade integrates both the optimistic and the pessimistic performances of each alternative and is, therefore, more comprehensive than either of them. An illustrative example of a nuclear waste dump site selection shows that the compromised grade has a better discriminating power than the optimistic and pessimistic DEA models. We point out that the DEA models discussed in this paper are all based on the so-called weighted additive DEA models, which can be represented in the envelopment forms or the dual multiplier forms. In the envelopment forms, the priority weights of attributes are attached to the slack variables of the objective functions and in the multiplier forms, the upper weight bounds are imposed on the attribute weights.

Therefore, choosing a *priori* weights of attributes, using AHP, in the proposed models is an important matter. However, one of the problems that may occur in practical situations is the difficulty of gathering the different views from some experts for use in AHP. This restricts us in deriving the priority weights of attributes, which further should be used in the proposed additive DEA-based GRA models. At the same time, the equal weight assumption might not be acceptable for decision-makers. In such situations, variable (data-dependent) weights are recommended like range-adjusted weights set introduced in Cooper *et al.* (1999) or a slacks-based distance function proposed by Tone (2001). Finally, the further studies may use those combined AHP and DEA methodologies in conjunction with GRA, which do not necessarily impose weight bounds on the attribute weights. The interested readers may refer to the following papers: Ho and Oh (2010), Jablonsky (2007), Sinuany-Stern *et al.* (2000) for ranking the efficient/inefficient units in DEA models using AHP in a two-stage process. Chen (2002), Cai and Wu (2001), Feng *et al.* (2004), Kim (2000), Pakkar (2014) for weighting the inputs and outputs in the DEA structure using AHP and (Liu and Chen, 2004) for constructing a convex combination of weights using AHP and DEA.

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Appendix

We assume that $e_j = 0$ and $w_0 = 0$ in model (11). In addition, without loss of generality, we assume that the first alternative has the highest grey relational coefficient within a group of alternatives for the first attribute, that is $\xi_{11} = \max\{\xi_{i1}, i = 1, 2, \ldots, m\}$. Obviously $w_1 = 1/\zeta_{11}, w_2 = \ldots = w_n = 0$ is a feasible solution to model (11) for the first alternative. As $w_1\zeta_{11} + w_2\zeta_{12} + \ldots + w_n\zeta_{1n} - w_0 = 1$ the set of weights is also an optimal solution to model (11) for the first alternative. If the set of weights is used, the first alternative will always obtain a grey relational grade of 1.

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